## II B.Tech I Semester Regular Examinations, November 2007 SIGNALS AND SYSTEMS

## ( Common to Electronics \& Communication Engineering, Electronics \& Instrumentation Engineering, Bio-Medical Engineering, Electronics \& Control Engineering and Electronics \& Telematics)

Time: 3 hours
Max Marks: 80

## Answer any FIVE Questions <br> All Questions carry equal marks

1. (a) Write short notes on "Orthogonal Vector Space".
(b) A rectangular function $\mathrm{f}(\mathrm{t})$ is defined by
$f(t)=\left\{\begin{array}{cc}1 & (0<t<\Pi) \\ -1 & (\Pi<t<2 \Pi)\end{array}\right.$
Approximate the above function by a finite series of Sinusoidal functions.
2. (a) Prove that $\operatorname{Sinc}(o)=1$ and plot Sinc function.
(b) Determine the Fourier series representation of that Signal $x(t)=3 \operatorname{Cos}(\Pi t / 2$ $+\Pi / 4)$ using the method of inspection.
3. (a) Find the Fourier Transform of the signal shown figure 3a.


Figure 3a
(b) Find the Fourier Transform of the signal given below
$y(t)\left\{\begin{array}{cc}\cos 10 t, & -2 \leq t \leq 2 \\ 0, & \text { otherwise }\end{array}\right.$
4. (a) Explain how input and output signals are related to impulse response of a LTI system.
(b) Let the system function of a LTI system be $\frac{1}{j w+2}$. What is the output of the system for an input $(0.8)^{t} u(t)$.
5. (a) State and Prove Properties of auto correlation function?
(b) A filter has an impulse response $h(t)$ as shown in figure 5b The input to the network is a pulse of unit amplitude extending from $t=0$ to $t=2$. By graphical means determine the output of the filter.


Figure 5b
6. (a) Consider the signal $x(t)=\left(\frac{\sin 50 \Pi t}{\Pi t}\right)^{2}$ which to be sampled with a sampling frequency of $\omega_{s}=150 \Pi$ to obtain a signal $\mathrm{g}(\mathrm{t})$ with Fourier transform $\mathrm{G}(\mathrm{j} \omega)$. Determine the maximum value of $\omega_{0}$ for which it is guaranteed that $G(j \omega)=75 \times(j \omega)$ for $|\omega| \leq \omega_{0}$ where $X(j \omega)$ is the Fourier transform of $\mathrm{x}(\mathrm{t})$.
(b) The signal $x(t)=u\left(t+T_{0}\right)-u\left(t-T_{0}\right)$ can undergo impulse train sampling without aliasing, provided that the sampling period $\mathrm{T}<2 T_{0}$. Justify.
(c) The signal $\mathrm{x}(\mathrm{t})$ with Fourier transform $X(j \omega)=u\left(\omega+\omega_{0}\right)-u\left(\omega-\omega_{0}\right)$ can undergo impulse train sampling without aliasing, provided that the sampling period $T<\pi / \omega_{0}$. Justify.
$[6+5+5]$
7. (a) Obtain the inverse laplace transform of $\mathrm{F}(\mathrm{s})=\frac{1}{s^{2}(s+2)}$ by convolution integral.
(b) Using convolution theorem find inverse laplace transform of $\frac{s}{\left(s^{2}+a^{2}\right)^{2}}$.
(c) Define laplace transform of signal $\mathrm{f}(\mathrm{t})$ and its region of convergence. $[6+6+4]$
8. (a) A finite sequence $\mathrm{x}[\mathrm{n}]$ is defined as $\mathrm{x}[\mathrm{n}]=\{5,3,-2,0,4,-3\}$ Find $\mathrm{X}[\mathrm{Z}]$ and its ROC.
(b) Consider the sequence $\mathrm{x}[\mathrm{n}]=\left\{\begin{array}{cc}a^{n} & 0 \leq n \leq N-1, a>0 \\ 0 & \text { otherwise }\end{array}\right.$ Find $\mathrm{X}[\mathrm{Z}]$.
(c) Find the Z-transform of $x(n)=\cos (n \omega) u(n)$.

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1. (a) Consider the pair of exponentially damped sinusoidal signals

$$
x_{1}(t)=A e^{\alpha t} \operatorname{Cos}(\omega t) \quad t \geq 0
$$

$$
x_{2}(t)=A e^{\alpha t} \sin (\omega t) \quad t \geq 0
$$

Assume that A, a and w are all real numbers,
the exponential damping factor $\alpha$ is negative and the frequency of oscillator $\omega$ is positive, the amplitude A can be positive or negative.
i. Derive the complex valued signal $\mathrm{x}(\mathrm{t})$ whose real part is $x_{1}(\mathrm{t})$ and imaginary part is $x_{2}(\mathrm{t})$.
ii. Determine $a(t)$ for $x(t)$ defined in part (i) where $a(t)$ is envelope of the complex signal which is given by
$a(t)=\sqrt{x_{1}^{2}(t)+x_{2}^{2}(t)}$
iii. How does the envelope $\mathrm{a}(\mathrm{t})$ vary with time t .
(b) Sketch the following signal $\mathrm{x}(\mathrm{t})=\mathrm{A}[\mathrm{u}(\mathrm{t}+\mathrm{a})-\mathrm{u}(\mathrm{t}-\mathrm{a})]$ for $\mathrm{a}>0$ Also determine whether the given signal is a power signal on an energy signal or neither.
(c) State the properties of even and odd functions.

$$
[6+6+4]
$$

2. (a) Write short notes on "Complex Fourier Spectrum".
(b) Find the Exponential Fourier series for the rectified Sine wave as shown in figure 2.

$$
[6+10]
$$



Figure 2
3. Find the Fourier Transform of the following function
(a) A single symmetrical Triangular Pulse
(b) A single symmetrical Gate Pulse
(c) A single cosine wave at $t=0$

$$
[8+4+4]
$$

4. (a) Explain the characteristics of an ideal LPF. Explain why it can't be realized.
(b) Differentiate between causal and non-causal systems.
5. (a) If $\mathrm{V}(\mathrm{t})=\operatorname{Sin} \omega_{o} \mathrm{t}$.
i. find $\mathrm{R}(\Gamma)$
ii. Find energy spectral density $G_{E}(\mathrm{f})=$ Fourier transform of $\mathrm{R}(\tau)$
(b) Applying the convolution theorem find Fourier Transform of $\left[A e^{-|a t|} \sin c 2 W t\right]$.
(c) Use the convolution theorem to find the spectrum of $\mathrm{x}(\mathrm{t})=\mathrm{A} \operatorname{Cos}^{2} \omega_{c} \mathrm{t}$

$$
[6+6+4]
$$

6. (a) A low pass signal $x(t)$ has a spectrum $x(f)$ given by
$x(f)=\begin{array}{cc}1-|f| / 200 & |f|<200 \\ 0 & \text { elsewhere }\end{array}$
Assume that $\mathrm{x}(\mathrm{t})$ is ideally sampled at $\mathrm{fs}=300 \mathrm{~Hz}$. Sketch the spectrum of $x_{\delta}(t)$ for $|f|<200$.
(b) The uniform sampling theorem says that a band limited signal $x(t)$ can be completely specified by its sampled values in the time domain. Now consider a time limited signal $\mathrm{x}(\mathrm{t})$ that is zero for $|t| \geq T$. Show that the spectrum $\mathrm{x}(\mathrm{f})$ of $\mathrm{x}(\mathrm{t})$ can be completely specified by the sampled values $\mathrm{x}\left(\mathrm{k} f_{o}\right)$ where $f_{0} \leq 1 / 2 T$.
7. (a) State the properties of the ROC of L.T.
(b) Determine the function of time $x(t)$ for each of the following laplace transforms and their associated regions of convergence.
[8+8]
i. $\frac{(s+1)^{2}}{s^{2}-s+1} \quad \operatorname{Re}\{S\}>1 / 2$
ii. $\frac{s^{2}-s+1}{(s+1)^{2}} \quad \operatorname{Re}\{S\}>-1$
8. (a) Find the Z-transform of $a^{n} \cos (n \omega) u(n)$
(b) Find the inverse Z-transform of $X(Z)=\frac{2+Z^{3}+3 Z^{-4}}{Z^{2}+4 Z+3} \quad|Z|>0$
(c) Find the Z-transform of the following signal with the help of linearity and shifting properties. $x(n)=\left\{\begin{array}{cc}1 & \text { for } 0 \leq N-1 \\ 0 & \text { elsewhere }\end{array}\right.$.

$$
[5+5+6]
$$

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1. (a) Explain orthogonality property between two complex functions $f 1(t)$ and $f 2(t)$ for a real variable $t$.
(b) Discuss how an unknown function $\mathrm{f}(\mathrm{t})$ can be expressed using infinite mutually orthogonal functions. Hence, show the representation of a waveform $f(t)$ using trigonometric fourier series.
2. (a) Derive polar Fourier series from the exponential Fourier series representation and hence prove that $D_{n}=2\left|C_{n}\right|$
(b) Show that the magnitude spectrum of every periodic function is Symmetrical about the vertical axis passing through the origin.
3. (a) Obtain the Fourier transform of the following functions:
i. Impulse function $\delta(\mathrm{t})$
ii. DC Signal
iii. Unit step function.
(b) State and prove time differentiation property of Fourier Transform. [12+4]
4. (a) Explain how input and output signals are related to impulse response of a LTI system.
(b) Let the system function of a LTI system be $\frac{1}{j w+2}$. What is the output of the system for an input $(0.8)^{t} u(t)$.
[8+8]
5. (a) A signal $\mathrm{y}(\mathrm{t})$ given by $y(t)=C_{0}+\sum_{n=1}^{\infty} C_{n} \cos \left(n \omega_{o} t+\theta_{n}\right)$. Find the auto correlation and PSD of $y(t)$.
(b) Find the mean square value (or power) of the output voltage $y(t)$ of the system shown in figure 5 b. If the input voltage PSD. $S_{2}(\omega)=\operatorname{rect}(\omega / 2)$. Calculate the power (mean square value) of input signal $x(t)$.
$[8+8]$


Figure 5b
6. (a) Consider the signal $x(t)=\left(\frac{\sin 50 \Pi t}{\Pi t}\right)^{2}$ which to be sampled with a sampling frequency of $\omega_{s}=150 \Pi$ to obtain a signal $\mathrm{g}(\mathrm{t})$ with Fourier transform $\mathrm{G}(\mathrm{j} \omega)$. Determine the maximum value of $\omega_{0}$ for which it is guaranteed that $G(j \omega)=75 \times(j \omega)$ for $|\omega| \leq \omega_{0}$ where $X(j \omega)$ is the Fourier transform of $\mathrm{x}(\mathrm{t})$.
(b) The signal $x(t)=u\left(t+T_{0}\right)-u\left(t-T_{0}\right)$ can undergo impulse train sampling without aliasing, provided that the sampling period $\mathrm{T}<2 T_{0}$. Justify.
(c) The signal $\mathrm{x}(\mathrm{t})$ with Fourier transform $X(j \omega)=u\left(\omega+\omega_{0}\right)-u\left(\omega-\omega_{0}\right)$ can undergo impulse train sampling without aliasing, provided that the sampling period $T<\pi / \omega_{0}$. Justify.
$[6+5+5]$
7. (a) Obtain the inverse laplace transform of $\mathrm{F}(\mathrm{s})=\frac{1}{s^{2}(s+2)}$ by convolution integral.
(b) Using convolution theorem find inverse laplace transform of $\frac{s}{\left(s^{2}+a^{2}\right)^{2}}$.
(c) Define laplace transform of signal $\mathrm{f}(\mathrm{t})$ and its region of convergence. $[6+6+4]$
8. (a) Find the Z-transform $\mathrm{X}(\mathrm{n})$.
i. $x[n]=\left(\frac{1}{2}\right)^{n} u[n]+\left(\frac{1}{3}\right)^{n} u[n]$
ii. $x[n]=\left(\frac{1}{3}\right)^{n} u[n]+\left(\frac{1}{2}\right)^{n} u[-n-1]$
(b) Find inverse z transform of $\mathrm{x}(\mathrm{z})$ using long division method $\mathrm{x}(\mathrm{z})=\frac{2+3 \mathrm{z}^{-1}}{\left(1+\mathrm{z}^{-1}\right)\left(1+0.25 \mathrm{z}^{-1}-\frac{z^{-2}}{8}\right)}$

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1. (a) Define
i. Basis Functions
ii. Norm.
(b) Determine whether each of the following sequences are periodic or not. If periodic determine the fundamental period.
i. $x_{1}(\mathrm{n})=\sin (6 \pi \mathrm{n} / 7)$
ii. $x_{2}(\mathrm{n})=\operatorname{Sin}(\mathrm{n} / 8)$
(c) Consider the rectangular pulse $\mathrm{x}(\mathrm{t})$ of unit amplitude and a duration of 2 time units depicted in figure 1c.

$$
[8+4+4]
$$



Figure 1c
Sketch $y(t)=x(2 t+3)$.
2. (a) Derive polar Fourier series from the exponential Fourier series representation and hence prove that $D_{n}=2\left|C_{n}\right|$
(b) Show that the magnitude spectrum of every periodic function is Symmetrical about the vertical axis passing through the origin.
3. (a) Obtain the Fourier transform of the following functions:
i. Impulse function $\delta(\mathrm{t})$
ii. DC Signal
iii. Unit step function.
(b) State and prove time differentiation property of Fourier Transform. [12+4]
4. (a) Explain the difference between causal and non-causal systems.
(b) Consider a stable LTI system that is characterized by the differential equation $\frac{d^{2} y(t)}{d t^{2}}+4 \frac{d y(t)}{d t}+3 y(t)=\frac{d x(t)}{d t}+2 x(t)$
Find its response for input $x(t)=e^{-t} u(t)$.
5. (a) A waveform $\mathrm{m}(\mathrm{t})$ has a Fourier transform $\mathrm{M}(\mathrm{f})$ whose magnitude is as shown in figure 5a. Find the normalized energy content of the waveform.


Figure 5a
(b) The signal $\mathrm{V}(\mathrm{t})=\cos \omega_{0} \mathrm{t}+2 \sin 3 \omega_{0} \mathrm{t}+0.5 \sin 4 \omega_{0} \mathrm{t}$ is filtered by an RC low pass filter with a 3 dB frequency. $f_{c}=2 f_{0}$. Find the output power $S_{o}$.
(c) State parseval's theorem for energy X power signals.

$$
[6+6+4]
$$

6. (a) A signal $\mathrm{x}(\mathrm{t})=2 \cos 400 \pi \mathrm{t}+6 \cos 640 \pi \mathrm{t}$. is ideally sampled at $f_{s}=500 \mathrm{~Hz}$. If the sampled signal is passed through an ideal low pass filter with a cut off frequency of 400 Hz , what frequency components will appear in the output.
(b) A rectangular pulse waveform shown in figure 6 b is sampled once every $T_{S}$ seconds and reconstructed using an ideal LPF with a cutoff frequency of $f_{s} / 2$. Sketch the reconstructed waveform for $T_{s}=\frac{1}{6} \sec$ and $T_{s}=\frac{1}{12} \mathrm{sec}$.


Figure 6b
7. (a) Find inverse Laplace transform of the following:
i. $\frac{s^{2}+6 s+7}{s^{2}+3 s+2} \quad \operatorname{Re}(s)>-1$
ii. $\frac{s^{3}+2 s^{2}+6}{s^{2}+3 s} \quad \operatorname{Re}(s)>0$
(b) Find laplace transform of $\cos \omega t$.
8. (a) Find the inverse Z-transform of the following $\mathrm{X}(\mathrm{z})$.
i. $X(Z)=\log \left(\frac{1}{1-a z^{-1}}\right),|z|>|a|$
ii. $X(Z)=\log \left(\frac{1}{1-a^{-1} z}\right),|z|<|a|$
(b) Find the Z-transform $\mathrm{X}(\mathrm{n}) x[n]=\left(\frac{1}{2}\right)^{n} u[n]+\left(\frac{1}{3}\right)^{n} u[-n-1]$.

